Diffusion & Large Vision Models

Motivation

Forward / backwards process

Variational formulation

Training

Inference

Faster sampling

Stanford University

ICME

Problem formulation

Suppose we are given a **sample** of observations from p_{data}

Sample

New generation

Objective. Generate new images from a distribution of images p_{data}

Problem formulation

Suppose we are given a **sample** of observations from p_{data}

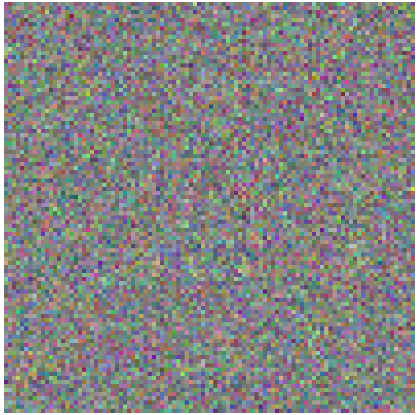
Sample

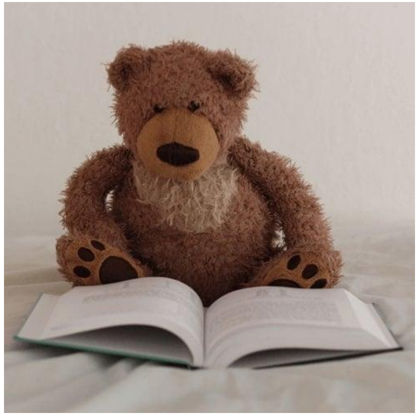
New generation

Objective. **Generate** new images from a distribution of images p_{data}

Unconditioned ← Focus of the 3 first lectures

Strategy: generate from noise





Why start from noise?

- Noise is **easy** to sample
- Introduces **randomness**
- Gaussian distribution has **nice properties**

Analogy: building a sculpture

The sculpture is already complete within the marble block, before I start my work. It is already there, I just have to chisel away the superfluous material.

–Michelangelo

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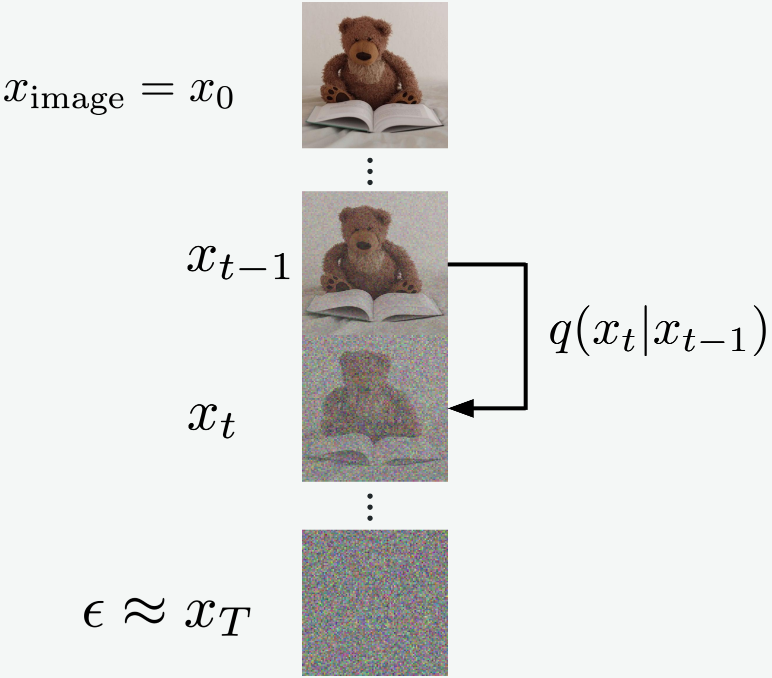
Intuition behind diffusion

1. Add **noise (forward process)**



Intuition behind diffusion

1. Add **noise (forward process)**

$x_{\text{image}} = x_0$

x_{t-1}

x_t

$\epsilon \approx x_T$

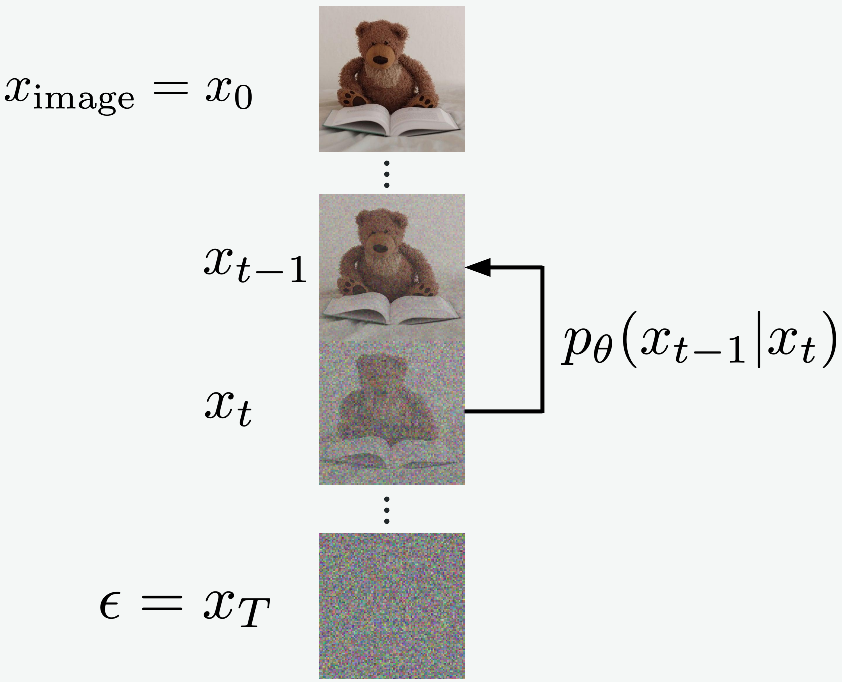
We know

$$q(x_t|x_{t-1})$$

Intuition behind diffusion

1. Add **noise (forward process)**

2. Learn to **denoise** it (**reverse process**)



Intuition behind diffusion

1. Add **noise (forward process)**

$x_{\text{image}} = x_0$

x_{t-1}

$q(x_t|x_{t-1})$

x_t

$\epsilon \approx x_T$

2. Learn to **denoise** it **(reverse process)**

$x_{\text{image}} = x_0$

We want to learn

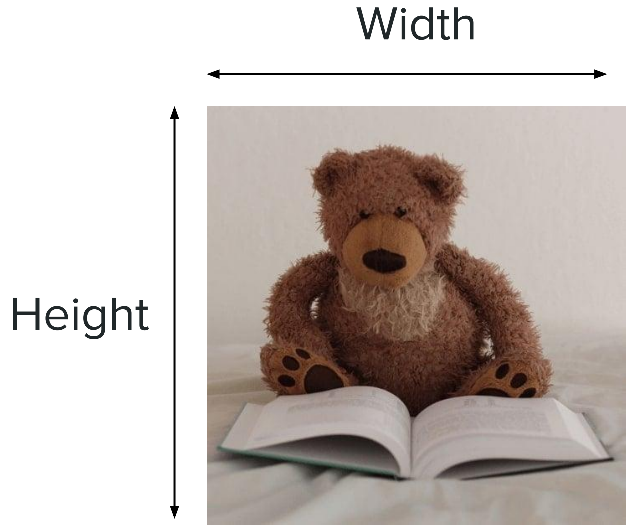
x_{t-1}

$p_\theta(x_{t-1}|x_t)$

x_t

$\epsilon = x_T$

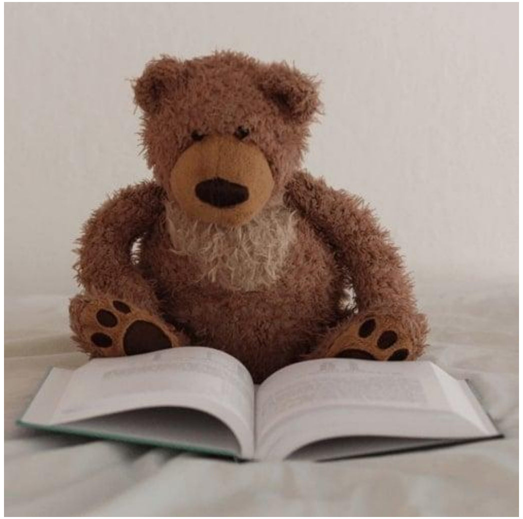
Image representation



R G B

pixel = (0 to 255, 0 to 255, 0 to 255)

Image representation



$$\longleftrightarrow \qquad x_{\text{image}} = \begin{pmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(n)} \end{pmatrix}$$

Probability distribution in 1-D space

$$x \sim \mathcal{N}(\mu, \sigma^2)$$

Scalar

Expectation

Variance

Probability distribution in high-dim space

$$x \sim \mathcal{N}(\mu, \Sigma)$$

Vector

$$\begin{pmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(n)} \end{pmatrix}$$

Expectation

$$\begin{pmatrix} \mu^{(1)} \\ \mu^{(2)} \\ \vdots \\ \mu^{(n)} \end{pmatrix}$$

Covariance

$$\begin{pmatrix} \Sigma^{(1,1)} & \Sigma^{(1,2)} & \cdots & \Sigma^{(1,n)} \\ \Sigma^{(2,1)} & \Sigma^{(2,2)} & \cdots & \Sigma^{(2,n)} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma^{(n,1)} & \Sigma^{(n,2)} & \cdots & \Sigma^{(n,n)} \end{pmatrix}$$

Meaning of probability distribution in high-dim space

$$x \sim \mathcal{N}(\mu, \Sigma)$$

$$x^{(i)} \sim \mathcal{N}(\mu^{(i)}, \Sigma^{(i,i)}) \quad \text{and} \quad \mathrm{Cov}(x^{(i)}, x^{(j)}) = \Sigma^{(i,j)}$$

Good news. We will mainly (if not only) be dealing with **isotropic** Gaussians:

$$\Sigma = \sigma^2 I = \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{pmatrix}$$

Meaning of probability distribution in high-dim space

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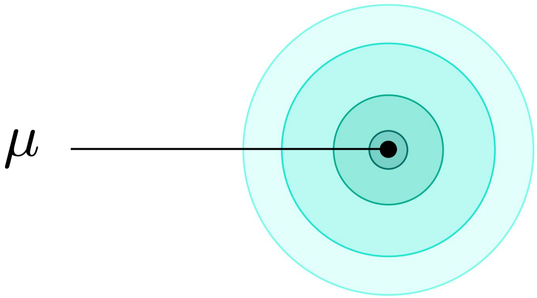

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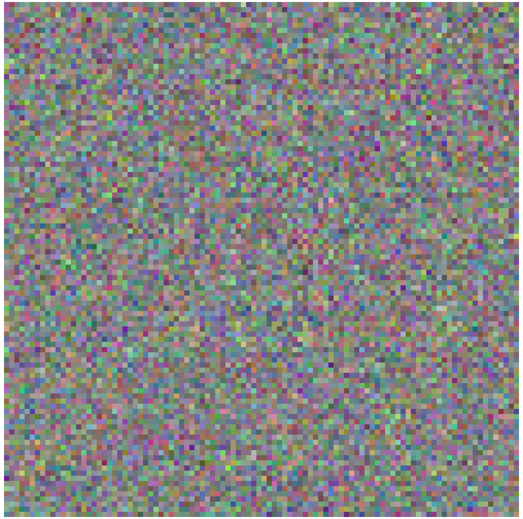
Good news. We will mainly (if not only) be dealing with **isotropic** Gaussians:

$$p(x) = \frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{\|x-\mu\|^2}{2\sigma^2}\right)$$

probability density
is known and "easy"

Noise representation

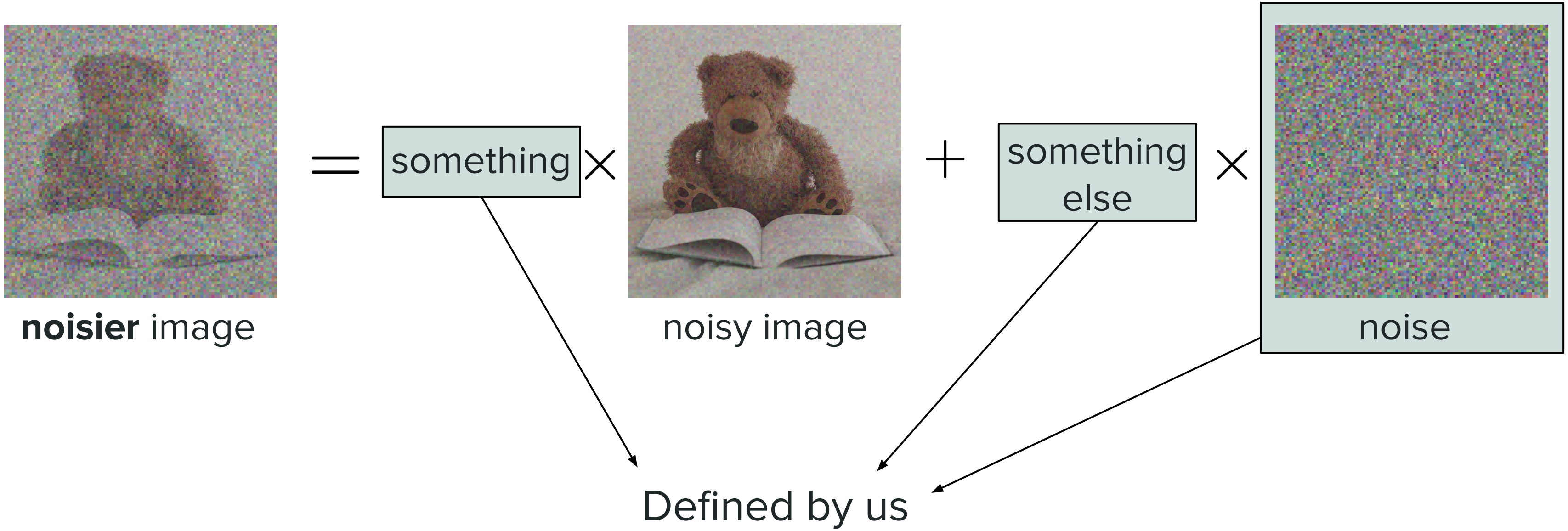


$$\longleftrightarrow \quad \epsilon = \begin{pmatrix} \epsilon^{(1)} \\ \epsilon^{(2)} \\ \vdots \\ \epsilon^{(n)} \end{pmatrix} \sim \mathcal{N}(0, I)$$

In other words, $\epsilon^{(i)} \sim \mathcal{N}(0, 1)$ independent and identically distributed

Process behind this

q is **chosen** by us and its **distribution** is **known** (reminder: it is stochastic!)



Process behind this

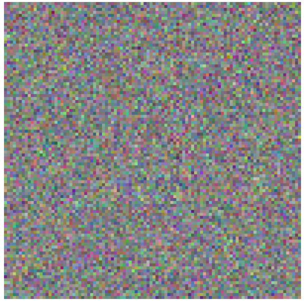
q is **chosen** by us and its **distribution** is **known** (reminder: it is stochastic!)

noisier image

= something ×

noisy image

+ something else ×

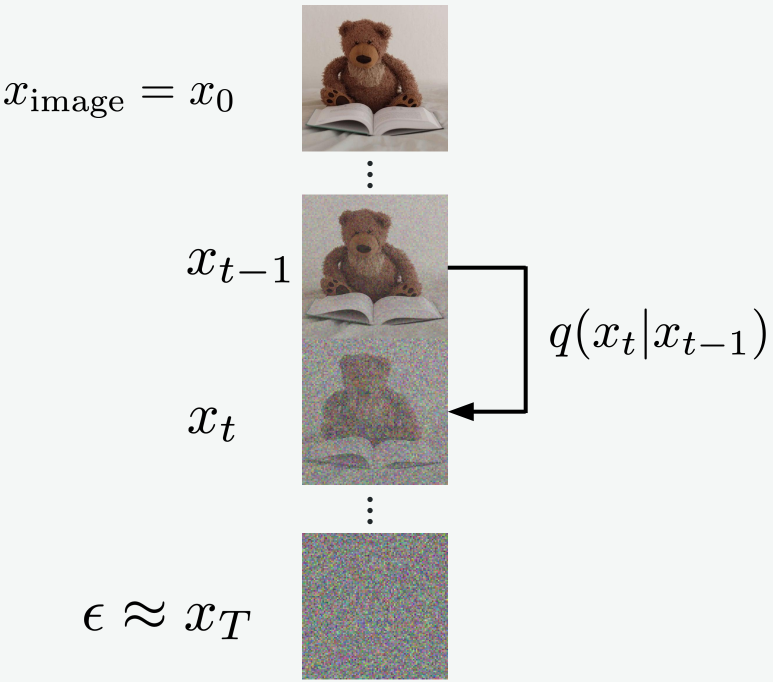


noise

"**Noise schedule**"

Mathematical formalism of forward process

1. Add **noise (forward process)**



Noisier image

Noise

$$q(x_t|x_{t-1}) = \mathcal{N}(\sqrt{1-\beta_t}x_{t-1}, \beta_t I)$$

Previous image

with $0 \leqslant \beta_1 < \beta_2 < \ldots < \beta_T \leqslant 1$

noise schedule

Mathematical formalism of forward process

$$q(x_t|x_{t-1}) = \mathcal{N}(\sqrt{1-\beta_t}x_{t-1}, \beta_t I)$$

$$\downarrow$$

$$\boxed{q(x_t|x_0) = \mathcal{N}(\sqrt{\overline{\alpha}_t}x_0, (1-\overline{\alpha}_t)I)}$$

with $\bar{\alpha}_t = \prod_{i=1}^{t} \alpha_i$ and $\alpha_t = 1 - \beta_t$

Derivation: Variance of sum of independent Gaussians is sum of respective variances.

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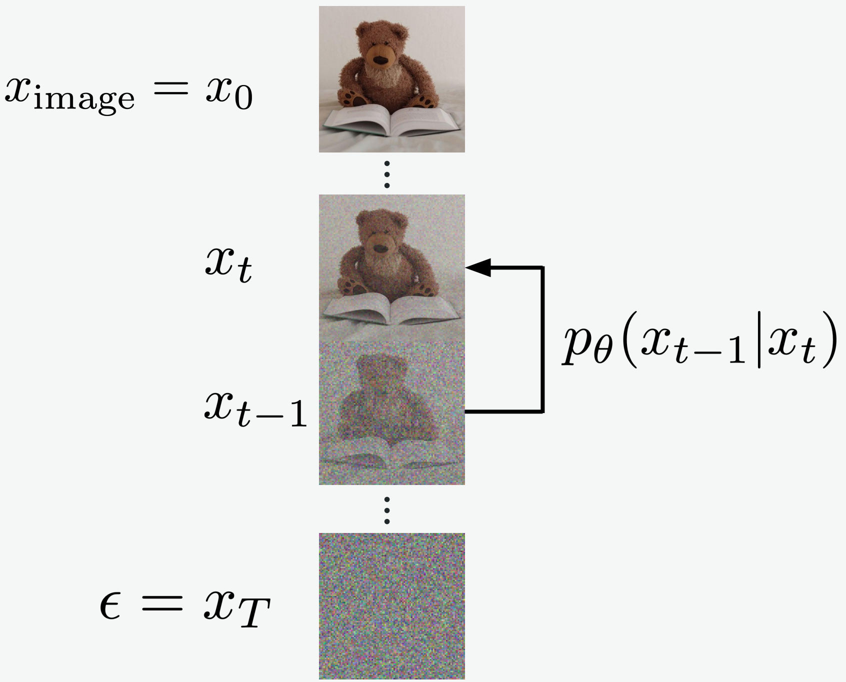
Learn reverse process

2. Learn to **denoise** it (**reverse process**)



Learn reverse process

Objective. We want to learn:

$$p_\theta$$

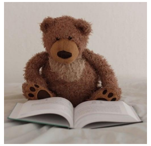
Objective

$$\max_{\theta} \qquad p_\theta(x_0)$$

find parameters θ that maximize the quantity

probability given by the model of parameters θ

training images



Objective

computationally stable +

nice properties from log

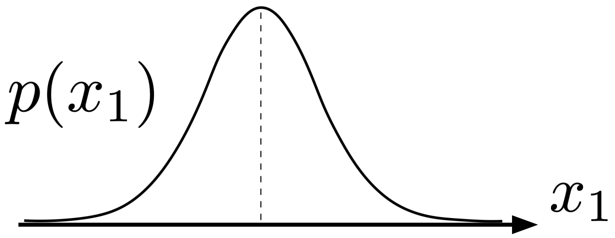
$$\max_{\theta} \log p_{\theta}(x_0)$$

find parameters θ that maximize the quantity

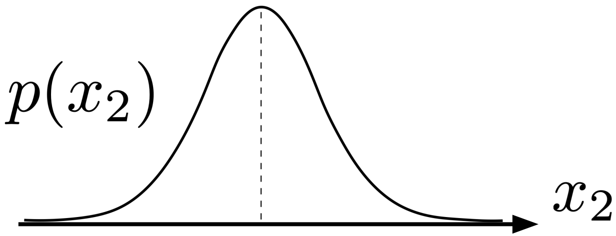
probability given by the model of parameters θ

training images

Refresher: joint probability distribution



Probability density of x_1



Probability density of x_2

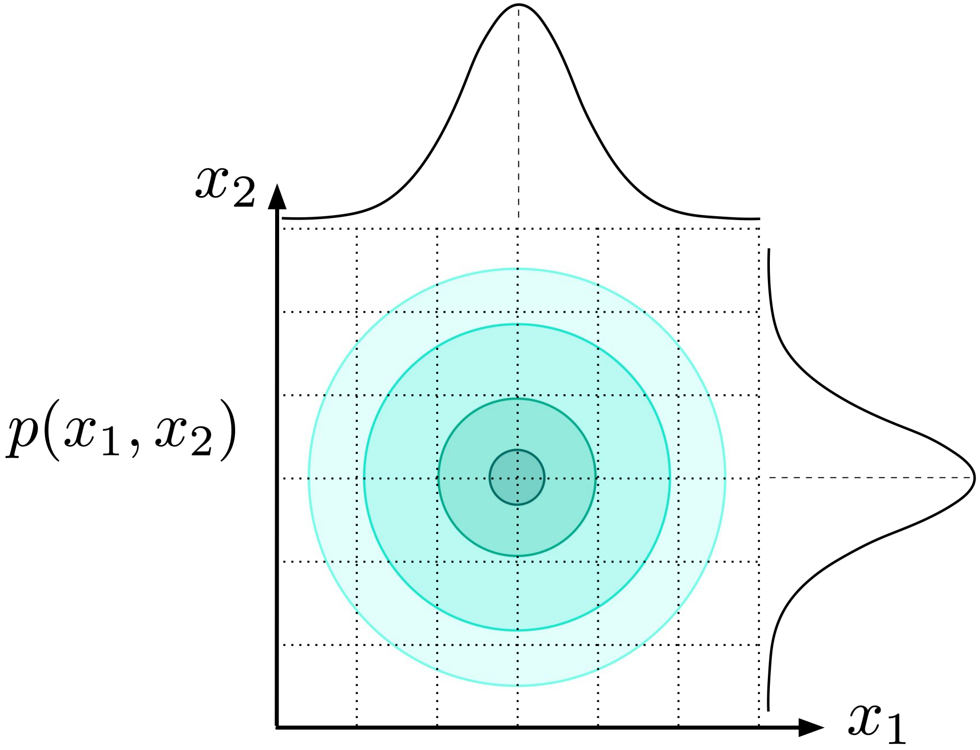
Refresher: joint probability distribution

Joint probability distribution.

$$p(x_1, x_2) = p(x_1) \times p(x_2|x_1)$$

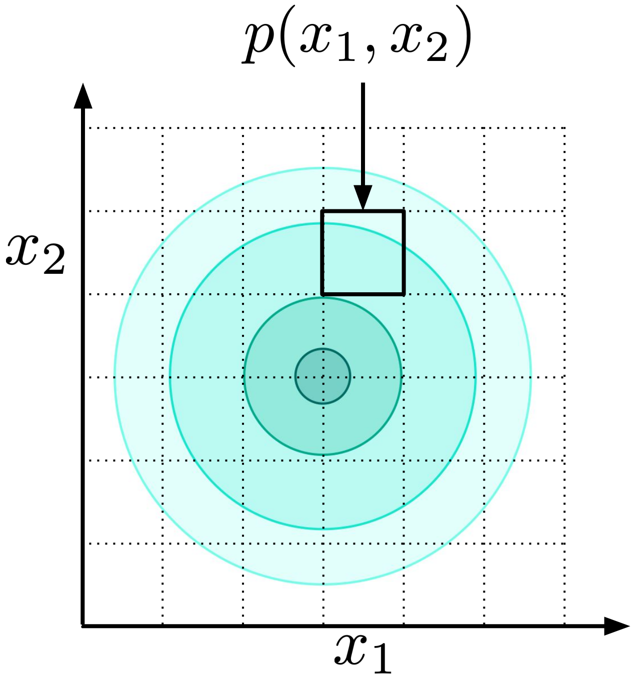
Conditional probability: if you know x_1 then what is the probability of x_2

Refresher: joint probability distribution



Joint probability $p(x_1, x_2)$ is the probability that we have x_1 **and** x_2

Refresher: joint probability distribution



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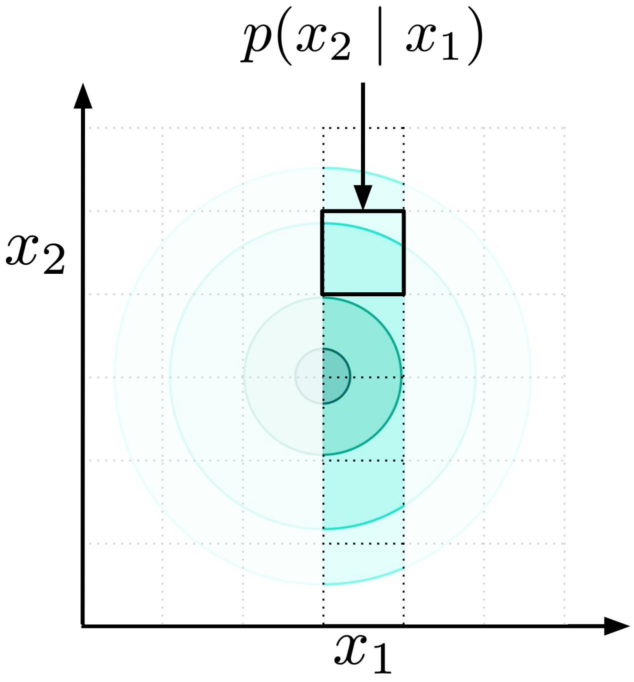
Refresher: joint probability distribution



Conditional probability $p(x_2 \mid x_1)$

is the probability that we have x_2

knowing that we have x_1

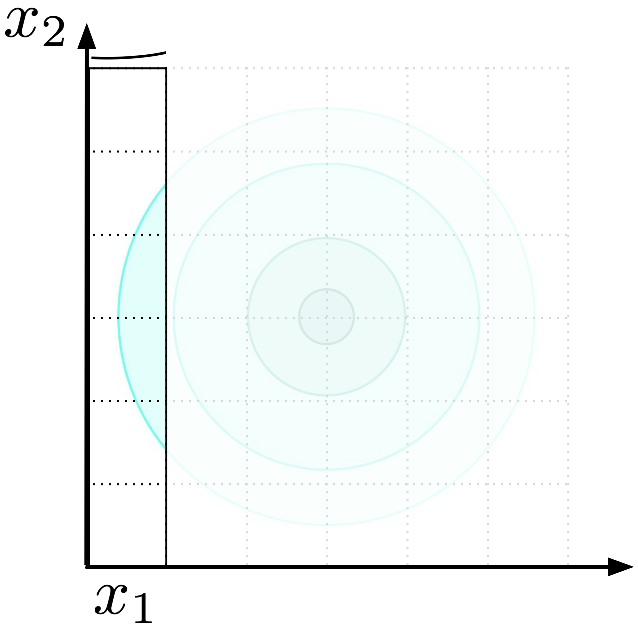
Refresher: joint probability distribution

Marginalization.

$$p(x_1) = \int p(x_1, x_2) dx_2$$

Sum across all x_2

Refresher: joint probability distribution



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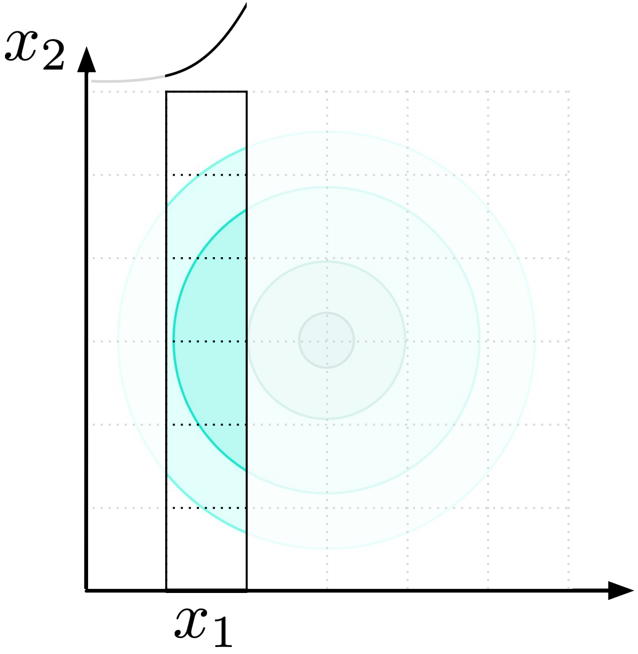
Refresher: joint probability distribution



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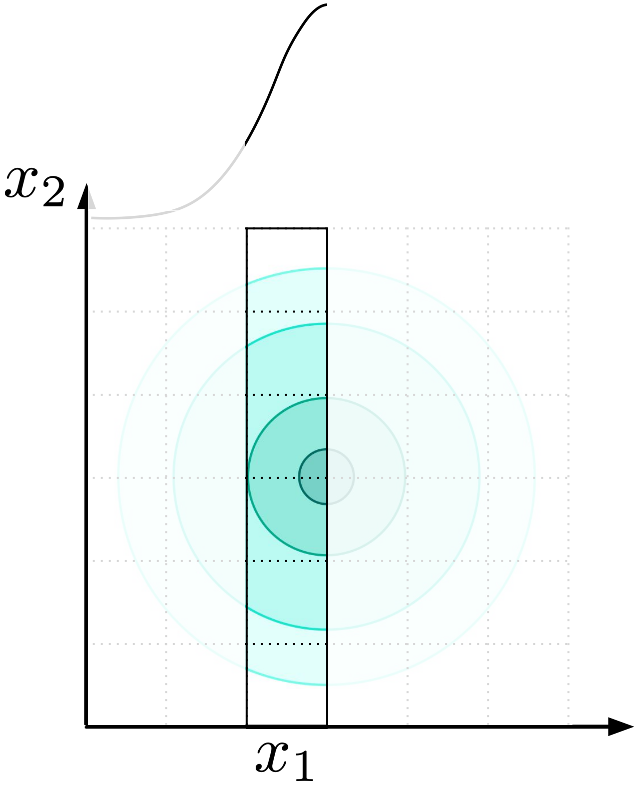

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Sum across all x_2

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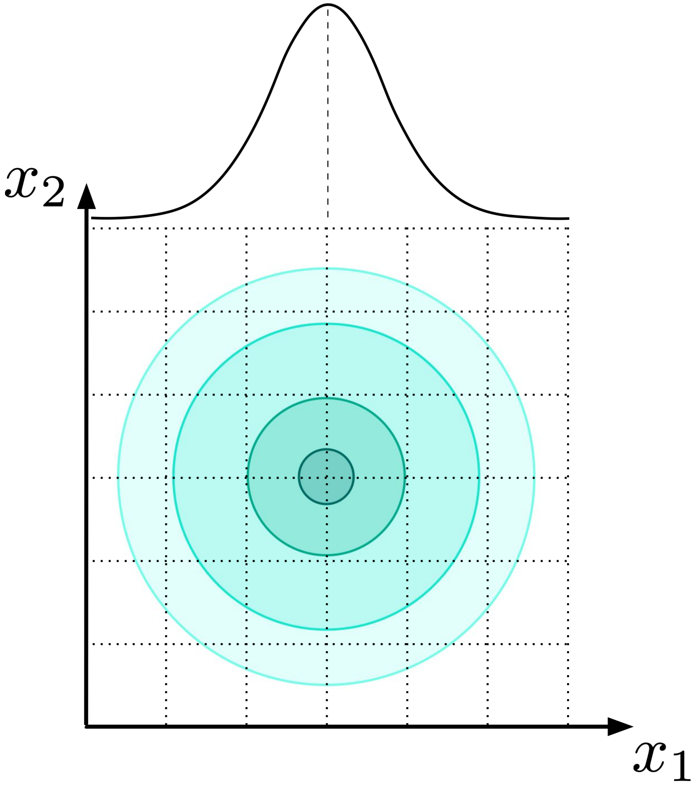


$$p(x_1) = \int p(x_1, x_2) dx_2$$

Sum across all x_2

Refresher: joint probability distribution

Joint probability distribution.

$$p(x_1, x_2, ..., x_T) = p(x_1) \times p(x_2|x_1) \times ... \times p(x_T|x_1, ..., x_{T-1})$$

Marginalization.

$$p(x_1) = \int p(x_1, x_2, ..., x_T) dx_2 ... dx_T$$

Refresher: joint probability distribution

Joint probability distribution.

$$p(x_1, x_2, \dots, x_T) = p(x_1) \times p(x_2|x_1) \times \dots \times p(x_T|x_1, \dots, x_{T-1})$$

$$p(x_1, x_2, \dots, x_T) = p(x_{1:T})$$

Marginalization.

$$p(x_1) = \int p(x_1, x_2, \dots, x_T) dx_2 \dots dx_T$$

Refresher: joint probability distribution

Joint probability distribution (Simplified notation).

$$p(x_{1:T}) = p(x_1)\prod_{t=2}^{T} p(x_t|x_{1:t-1})$$

Marginalization (Simplified notation).

$$p(x_1) = \int p(x_{1:T})dx_{2:T}$$

Problem

$$\log p_\theta(x_0) = \ ?$$

Derivation of a tractable loss

Strategy.

1. Derive **lower bound** for maximum (log-)likelihood estimation
2. **Expand terms** of lower bound
3. Show lower bound is **tractable**
4. Deduce **final loss function**

Step 1: Derivation of a lower bound

1 Derive **lower bound** for maximum (log-)likelihood estimation

$$\mathbb{E}_{x_0 \sim q(x_0)}\left[\log p_\theta(x_0)\right] \geqslant \boxed{\mathbb{E}_{x_{0:T} \sim q(x_{0:T})}\left[\log \frac{p_\theta(x_{0:T})}{q(x_{1:T}|x_0)}\right]}$$

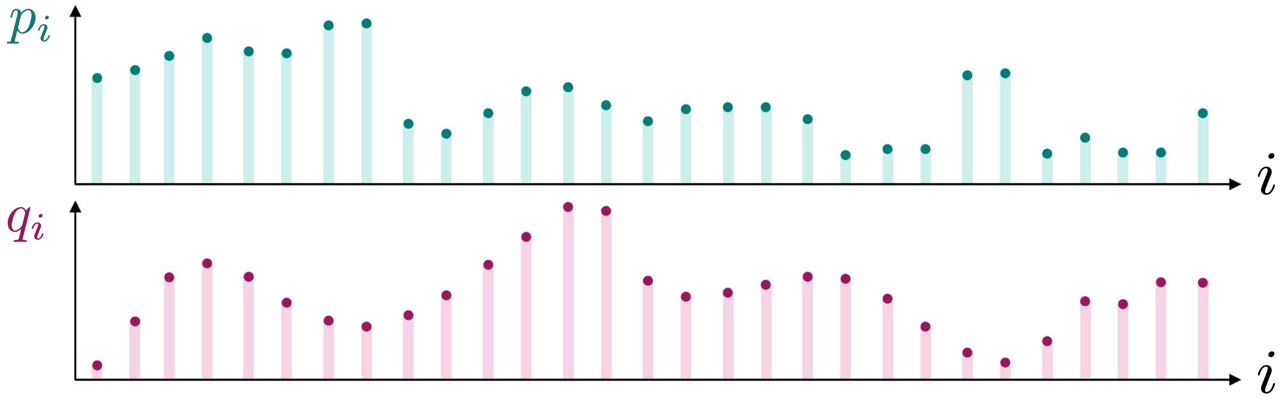
ELBO = **E**vidence **L**ower **BO**und

Derivation: Use Jensen's inequality on a convenient variational distribution

Full derivation in Denoising Diffusion Probabilistic Models, Ho et al., 2020.

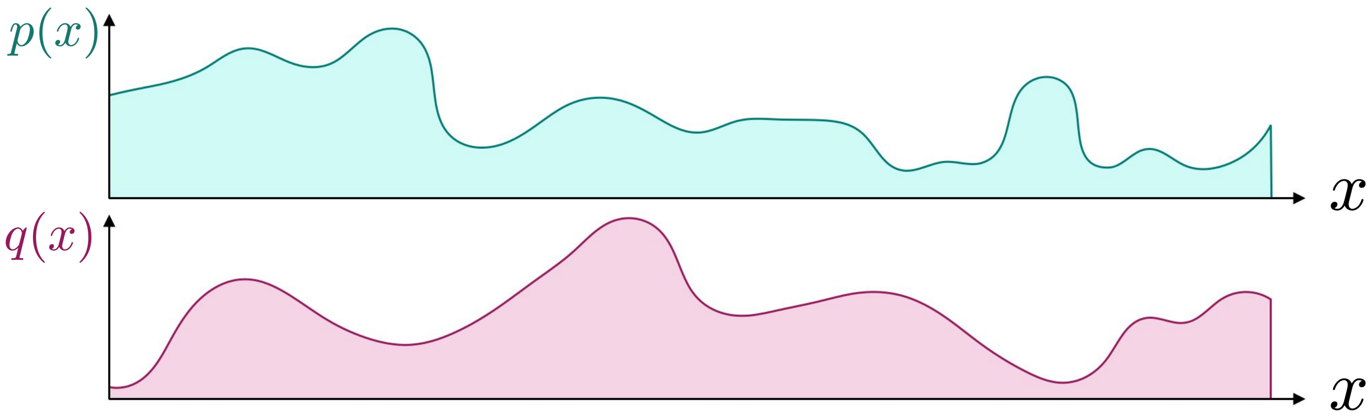
Refresher: KL divergence (discrete)

$$\mathrm{KL}(P\|Q)=\sum_{i=1}^{n}p_i\log\left(\frac{p_i}{q_i}\right)$$



Refresher: KL divergence (continuous)

$$\mathrm{KL}(P\|Q) = \int \log\left(\frac{p(x)}{q(x)}\right) p(x)\, dx = \mathbb{E}_{x\sim p}\left[\log\left(\frac{p(x)}{q(x)}\right)\right]$$



Step 2: Expand terms of the lower bound

2 **Expand terms** of lower bound

$$\text{ELBO} = -\sum_{t=2}^{T} \text{KL}\big(q(x_{t-1} \mid x_t, x_0) \,\|\, p_\theta(x_{t-1} \mid x_t)\big) + \text{some other terms}$$

Can we compute this?

We want to learn θ

Derivation: Use properties of the log function and rearrange terms

Full derivation in Denoising Diffusion Probabilistic Models, Ho et al., 2020.

Step 3: Show lower bound is tractable

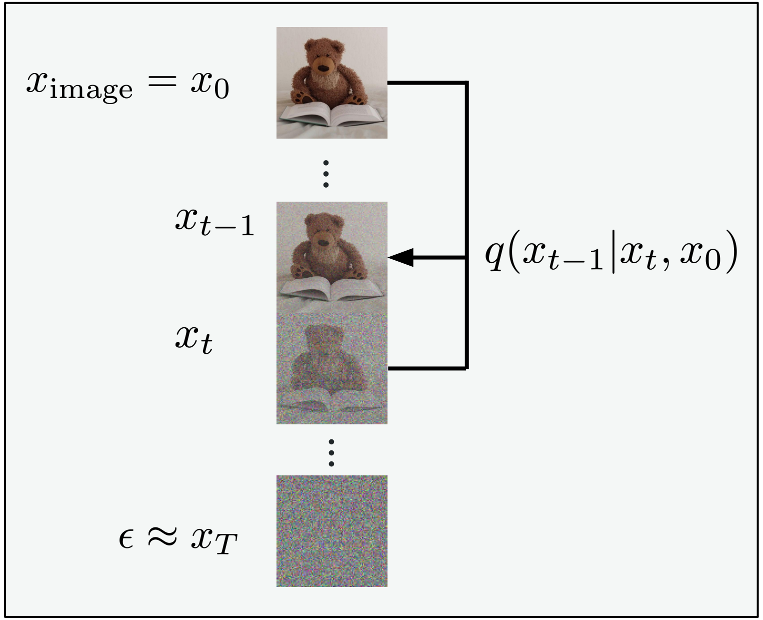
3 Show lower bound is **tractable**

a Show $q(x_{t-1} \mid x_t, x_0)$ is tractable

$$\text{ELBO} = -\sum_{t=2}^{T} \text{KL}\left(q(x_{t-1} \mid x_t, x_0) \,\|\, p_\theta(x_{t-1} \mid x_t)\right) + \text{some other terms}$$

Step 3: Show lower bound is tractable

3 a Show $q(x_{t-1} \mid x_t, x_0)$ is tractable



noisy image

clean image

$$q(x_{t-1}|x_t, x_0)$$

less noisy image

Refresher: Bayes' rule

Likelihood

Posterior $$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$ Prior

Marginal likelihood, aka evidence

Step 3: Show lower bound is tractable

3 a Show $q(x_{t-1} \mid x_t, x_0)$ is tractable

$$\boxed{q(x_{t-1}|x_t, x_0) = \mathcal{N}(\tilde{\mu}_t, \tilde{\beta}_t)}$$

sampled noise used to go from clean to noisy image

with $\tilde{\mu}_t = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\overline{\alpha}_t}}\epsilon_t\right)$

noisy image

Derivation: Compute density with Bayes' rule via $q(. \mid x_0)$ + Markov property

Step 3: Show lower bound is tractable

3 Show lower bound is **tractable**

a Show $q(x_{t-1} \mid x_t, x_0)$ is tractable

b Show $p_\theta(x_{t-1} \mid x_t)$ is tractable

$$\text{ELBO} = -\sum_{t=2}^{T} \text{KL}\big(q(x_{t-1} \mid x_t, x_0) \,\|\, p_\theta(x_{t-1} \mid x_t)\big) + \text{some other terms}$$

Step 3: Show lower bound is tractable

3 b Show $p_\theta(x_{t-1} \mid x_t)$ is tractable

We assume that p_θ is Gaussian:

$$\boxed{p_\theta(x_{t-1} \mid x_t) = \mathcal{N}\big(\mu_\theta(x_t, t), \Sigma_\theta(x_t, t)\big)}$$

Justification: If $q(x_t|x_{t-1})$ Gaussian, then $q(x_{t-1}|x_t)$ is approx. Gaussian

Full derivation in Denoising Diffusion Probabilistic Models, Ho et al., 2020.

Step 4: Deduce final loss function

4 Deduce **loss function**

$$\mathcal{L}_{\mathrm{DDPM}} = \mathbb{E}_{t, x_0, \epsilon}\left[\left\| \epsilon_\theta\left(\sqrt{\bar{\alpha}_t}\, x_0 + \sqrt{1-\bar{\alpha}_t}\, \epsilon,\ t\right) - \epsilon \right\|^2\right]$$

$t \sim \mathcal{U}\{1, T\}$ $\quad x_0 \sim q_0(x_0)$ $\quad \epsilon \sim \mathcal{N}(0, I)$

Derivation: Compute KL divergence between two Gaussians

Full derivation in Denoising Diffusion Probabilistic Models, Ho et al., 2020.

Summary of what we have done

Strategy.

1. Derive **lower bound** for maximum (log-)likelihood estimation ⟵ **ELBO!**
2. **Expand terms** of lower bound ⟵ **Surfaced KL divergence**
3. Show lower bound is **tractable** ⟵ **Bayes' rule + Gaussian properties**
4. Deduce **final loss function** ⟵ **Incredibly simple noise prediction!**

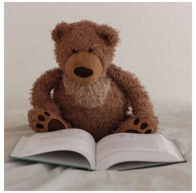
Diffusion & Large Vision Models

Training recipe

1. Sample:

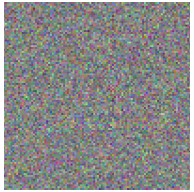
clean image

$x_0 \sim q_0(x_0)$



noise

$\epsilon \sim \mathcal{N}(0, I)$



time step

$t \sim \mathcal{U}\{1, T\}$

noised image

$$x_t = \sqrt{\overline{\alpha}_t} x_0 + \sqrt{1 - \overline{\alpha}_t} \epsilon$$

Training recipe

1. Sample:

clean image $x_0 \sim q_0(x_0)$

noise $\epsilon \sim \mathcal{N}(0, I)$

time step $t \sim \mathcal{U}\{1, T\}$

noised image

$$x_t = \sqrt{\overline{\alpha}_t} x_0 + \sqrt{1 - \overline{\alpha}_t} \epsilon$$

2. Use x_t and t to **predict** ϵ via $\epsilon_\theta(x_t, t)$

Training recipe

1. Sample:

clean image

$x_0 \sim q_0(x_0)$

noise

$\epsilon \sim \mathcal{N}(0, I)$

time step

$t \sim \mathcal{U}\{1, T\}$

noised image

$x_t = \sqrt{\overline{\alpha}_t} x_0 + \sqrt{1 - \overline{\alpha}_t} \epsilon$

2. Use x_t and t to **predict** ϵ via $\epsilon_\theta(x_t, t)$

3. Compute **loss** $\mathcal{L} = ||\epsilon_\theta(x_t, t) - \epsilon||^2$ and **backpropagate** through ϵ_θ

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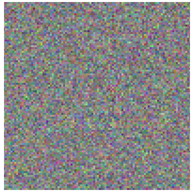
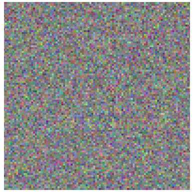
Inference recipe

1. Sample: **noise** $x_T \sim \mathcal{N}(0, I)$ 

2. Perform **iterative update** from x_t to x_{t-1}

Inference recipe

1. Sample: **noise** $x_T \sim \mathcal{N}(0, I)$



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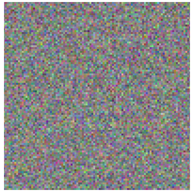
x_t

Inference recipe

1. Sample: **noise** $x_T \sim \mathcal{N}(0, I)$



2. Perform **iterative update** from x_t to x_{t-1}

x_t

$\epsilon_\theta(x_t, t)$

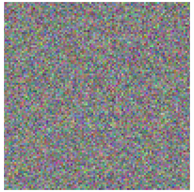
Predicted noise
to remove

Inference recipe

1. Sample:

noise

$x_T \sim \mathcal{N}(0, I)$



2. Perform **iterative update** from x_t to x_{t-1}

$$x_t - \frac{1 - \alpha_t}{\sqrt{1 - \overline{\alpha}_t}} \epsilon_\theta(x_t, t)$$

Predicted noise
to remove

Inference recipe

1. Sample:

noise

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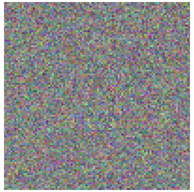

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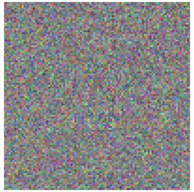

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$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\overline{\alpha}_t}}\epsilon_\theta(x_t, t)\right) + \sigma_t z$$

Predicted noise to remove

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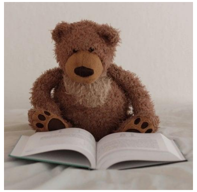
3. Obtain **final** image x_0



Landmark paper that is a must-read

DDPM = **D**enoising **D**iffusion **P**robabilistic **M**odels

Denoising Diffusion Probabilistic Models

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Abstract

We present high quality image synthesis results using diffusion probabilistic models, a class of latent variable models inspired by considerations from nonequilibrium thermodynamics. Our best results are obtained by training on a weighted variational bound designed according to a novel connection between diffusion probabilistic models and denoising score matching with Langevin dynamics, and our models naturally admit a progressive lossy decompression scheme that can be interpreted as a generalization of autoregressive decoding. On the unconditional CIFAR10 dataset, we obtain an Inception score of 9.46 and a state-of-the-art FID score of 3.17. On 256x256 LSUN, we obtain sample quality similar to ProgressiveGAN. Our implementation is available at https://github.com/hojonathanho/diffusion.

Denoising Diffusion Probabilistic Models, Ho et al., 2020.

Diffusion & Large Vision Models

Motivation

Forward / backwards process

Variational formulation

Training

Inference

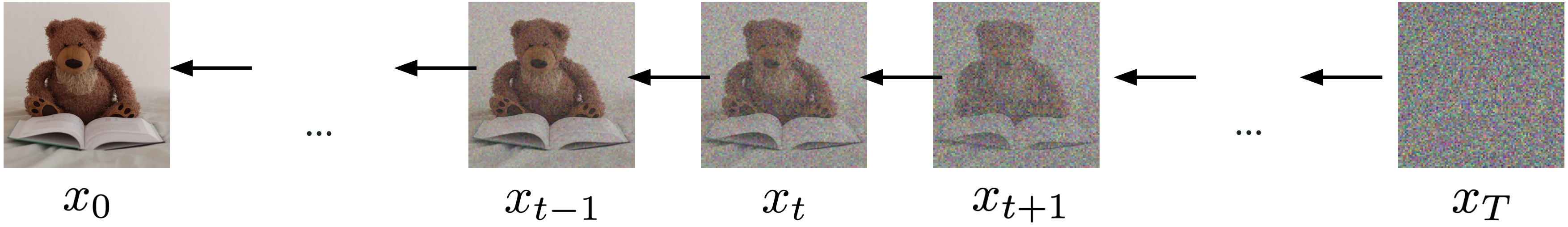
Faster sampling

Stanford University

ICME

Limitations

Generation process $p_\theta(x_{t-1} \mid x_t)$ $\quad T \gg 1$



hojonathanho/diffusion, official GitHub repository of the DDPM paper, 2020.

Limitations

Generation process $p_\theta(x_{t-1} \mid x_t)$ $T \gg 1$

Orders of magnitude

- $O(T)$ more expensive than VAE/GAN
- $T = 1000 \rightarrow$ possibly spend ~minutes per sample

hojonathanho/diffusion, *official GitHub repository of the DDPM paper, 2020.*

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Orders of magnitude

- $O(T)$ more expensive than VAE/GAN
- $T = 1000$ → possibly spend ~minutes per sample

In practice, DDPM is too slow at inference time!

Mitigation attempts

Starting point. Recall iterative generative formula between two steps:

$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t, t)\right) + \sigma_t z$$

Predicted noise to remove

Sampled from a Normal distribution

Mitigation attempts

Attempt. What if we derive a formula by induction?

$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t, t)\right) + \sigma_t z$$

$$x_{t-2} = \frac{1}{\sqrt{\alpha_{t-1}}}\left(x_{t-1} - \frac{1-\alpha_{t-1}}{\sqrt{1-\bar{\alpha}_{t-1}}}\epsilon_\theta(x_{t-1}, t-1)\right) + \sigma_{t-1} z$$

$$\vdots$$

$$x_{\tau_{i-1}} = Ax_{\tau_i} + \sum_{s=\tau_{i-1}+1}^{\tau_i} B_s\epsilon_\theta(x_s, s) + C_s z_s$$

Mitigation attempts

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$$\vdots$$

$$x_{\tau_{i-1}} = A x_{\tau_i} + \sum_{s=\tau_{i-1}+1}^{\tau_i} B_s \boxed{\epsilon_\theta(x_s, s)} + C_s z_s$$

Problem: Still need to evaluate function several times. No progress.

Mitigation attempts

Attempt 2. What if we directly skip steps?

$$\tau_0 = 0 < \tau_1 < \ldots < \tau_{S-1} < \tau_S = T$$

$$x_{\tau_{i-1}} = \sqrt{\frac{\bar{\alpha}_{\tau_{i-1}}}{\bar{\alpha}_{\tau_i}}} \left(x_{\tau_i} - \frac{1 - \bar{\alpha}_{\tau_i} / \bar{\alpha}_{\tau_{i-1}}}{\sqrt{1 - \bar{\alpha}_{\tau_i}}} \epsilon_\theta(x_{\tau_i}, \tau_i) \right) + \sigma_{\tau_i} z$$

Mitigation attempts

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Problem: mix large jumps + stochasticity = bad quality

Problem reformulation

Follow-up idea. How to remove stochasticity while making these larger jumps?

Key property. Loss function only relies on marginals $q(x_t \mid x_0)$

Problem statement. Find q such that:

- marginal is the same as in DDPM
- generation process can be made deterministic

Problem reformulation

$$q_\sigma(x_{t-1} \mid x_t, x_0) \to q_\sigma(x_t \mid x_0) = q(x_t \mid x_0)$$

$$q_\sigma(x_{t-1} \mid x_t, x_0) \to p_\theta(x_{t-1} \mid x_t)$$ can be made deterministic

$$q_\sigma(x_{t-1} \mid x_t, x_0) = \mathcal{N}\left(\sqrt{\bar{\alpha}_{t-1}}\, x_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \sigma_t^2}\, \frac{x_t - \sqrt{\bar{\alpha}_t}\, x_0}{\sqrt{1 - \bar{\alpha}_t}},\ \sigma_t^2 I\right)$$

Problem reformulation

$$q_\sigma(x_{t-1} \mid x_t, x_0)$$

$$q_\sigma(x_t \mid x_0) = q(x_t \mid x_0)$$

Justification: proof by induction

$p_\theta(x_{t-1} \mid x_t)$ can be made deterministic

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Problem reformulation

$$q_\sigma(x_{t-1} \mid x_t, x_0)$$

$$q_\sigma(x_t \mid x_0) = q(x_t \mid x_0)$$

Justification: proof by induction

$p_\theta(x_{t-1} \mid x_t)$ can be made deterministic

$$p_\theta(x_{t-1} \mid x_t) \triangleq q_{\sigma=0}(x_{t-1} \mid x_t, \hat{x}_0(x_t))$$

$$q_\sigma(x_{t-1} \mid x_t, x_0) = \mathcal{N}\left(\sqrt{\bar{\alpha}_{t-1}}\, x_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \cancel{\sigma_t^2}}\, \frac{x_t - \sqrt{\bar{\alpha}_t}\, x_0}{\sqrt{1 - \bar{\alpha}_t}}, \cancel{\sigma_t^2} I\right)$$

Why is DDIM called that way?

DDIM = **D**enoising **D**iffusion $\boxed{\textbf{I}\text{mplicit}}$ **M**odels

$$\sigma_t = 0$$

$$\hat{x}_0 = \frac{x_t - \sqrt{1-\bar{\alpha}_t}\,\epsilon_\theta(x_t, t)}{\sqrt{\bar{\alpha}_t}}$$

"our prediction of the clean image from time t"

Derivation: use the marginal distribution formula

Why is DDIM called that way?

DDIM = **D**enoising **D**iffusion $\boxed{\textbf{I}\text{mplicit}}$ **M**odels

$$\sigma_t = 0$$

$$\hat{x}_0 = \frac{x_t - \sqrt{1-\bar{\alpha}_t}\,\epsilon_\theta(x_t, t)}{\sqrt{\bar{\alpha}_t}}$$

$$x_{t-1} = \sqrt{\bar{\alpha}_{t-1}}\,\hat{x}_0 + \sqrt{1-\bar{\alpha}_{t-1}}\,\epsilon_\theta(x_t, t) \quad + \quad \varnothing$$

"Our prediction of the clean image from time t"

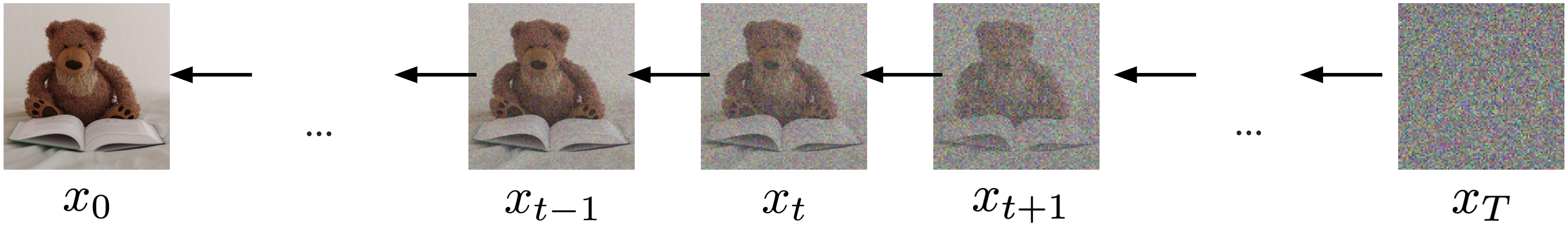
"...that we noise to time t-1..."

... with no stochasticity!

Why is the DDIM formulation useful?

Update rule

$$x_{t-1} = \sqrt{\bar{\alpha}_{t-1}}\,\hat{x}_0(t) + \sqrt{1-\bar{\alpha}_{t-1}}\,\epsilon_\theta(x_t, t)$$



Why is the DDIM formulation useful?

Update rule: accelerated!

$$x_{\tau_{i-1}} = \sqrt{\bar{\alpha}_{\tau_{i-1}}}\,\hat{x}_0(\tau_i) + \sqrt{1-\bar{\alpha}_{\tau_{i-1}}}\,\epsilon_\theta(x_{\tau_i}, \tau_i)$$

$$\tau_0 = 0 < \tau_1 < \ldots < \tau_{S-1} < \tau_S = T$$

$S \ll T$ with $\dfrac{T}{S} \triangleq$ "speedup"

Applications

Speedup

- Typically 10x-100x
- Need to trade off speed with quality

ermongroup/ddim, official GitHub repository of the DDIM paper, 2020.

Applications

Speedup

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Quality (based on CIFAR10 experiments)

Speed-up	1x	10x	20x	50x	100x
FID impact	baseline	+3%	+16%	+70%	+330%

Denoising Diffusion Implicit Models, *Song et al., 2020.*

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Discussion

- Skipping steps with inter-steps stochasticity $\rightarrow$ much worse quality
- Schedule for $\{\tau_i\}$ can be chosen as a hyperparameter

Denoising Diffusion Implicit Models, *Song et al., 2020.*

DDIM conclusion

Recipe

1. Choose "convenient" modeling assumptions compatible with DDPM optimization
2. Remove inter-step stochasticity = DDIM
3. Skip steps!

$$\boxed{p_\theta(x_{t-1} \mid x_t) \longrightarrow p_\theta(x_{\tau_{i-1}} \mid x_{\tau_i})}$$

Thank you for your attention!